MAT 112 Neidinger
Discrete Modeling III Homework: Function Notation and Cobweb graphs

It helps if graph sketches are drawn well. You may want to use graph paper or use a straight-edge to make a good graph-paper-like box. Then cobwebs can go parallel to these edges, vertical to function and horizontal to diagonal y=x.

1. Consider a population P_n with 50% growth each step but then a draw-down of 50.
   a) Write the recursive (one-step) formula for the population. Starting with $P_0 = 90$, compute the next three population to P_3.
   b) What is the function f(x) that is iterated in (a). Solve for the equilibrium.
   c) Sketch a cobweb diagram for population 0 to 200 (box, diagonal, and y=f(x), where you may extend the function outside the box). Starting with $P_0 = 90$, show steps that determine the next few populations and draw an arrow to show the direction of the steps. Repeat starting with $P_0 = 110$.
   d) Is the equilibrium stable or unstable?

2. This problem will iterate the function $f(x) = 2x(1-x)$.
   a) Write the recursive formula for $a_{n+1} = f(a_n)$. Starting with $a_0 = 0.9$, compute the next three values.
   b) Solve for the equilibrium values. (Multiply out and factor = 0 to get two solutions.)
   c) Sketch a cobweb diagram for values between 0 and 1 (big box, diagonal, and y=f(x) a parabola in this case). Starting with $a_0 = 0.9$, show next steps and draw an arrow to show the direction. Continue the steps ("the iteration") as long as you can see it on your diagram.
   d) Determine if each equilibrium is stable or unstable? Does the long term behavior (where the a_n values go) depend on where a_0 starts?

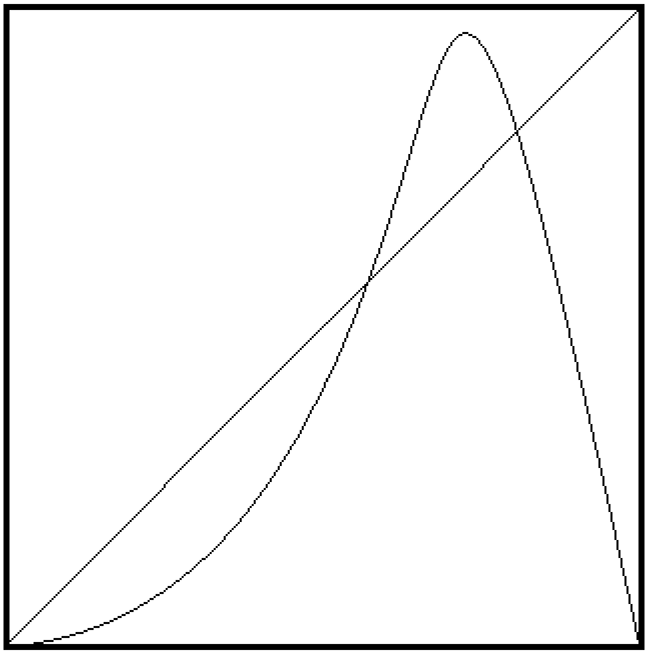
3. Consider a population P_n simply grows by 25% per step.
   a) Write the recursive formula for the population.
   b) What is the function f(x) that is iterated in (a). Solve for the equilibrium.
   c) Sketch a cobweb diagram for population 0 to 100 (box, diagonal, and y=f(x)). Starting with $P_0 = 10$, show steps including an arrow, continue as long as you can see it on your diagram.
   d) Is the equilibrium stable or unstable?

4. This problem will iterate the function $f(x) = -1.25x + 5$.
   a) Write the recursive formula for a_{n+1} in terms of a_n. Starting with $a_0 = 2$, compute the next three values to a_3.
   b) Solve for the equilibrium.
   c) Sketch a cobweb diagram for values between 0 and 5. Starting with $a_0 = 2$, show next steps and draw an arrow to show the direction. Continue the steps ("the iteration") as long as you can see it on your diagram.
   d) Is the equilibrium stable or unstable?

Over for 5.

Name ____________________________________

5. On the following sketch of $f(x)$, circle all equilibrium points for the iteration $a_{n+1} = f(a_n)$. Start at points near each equilibrium and draw a few cobweb steps to determine if the equilibrium is stable or unstable; label each as stable or unstable.



For any initial value (except the actual equilibrium points) in this domain, where will the iteration eventually end up? __________